Comparisons of Three Inversion Approaches for Recovering Gravity Anomalies From Altimeter Data

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Abstract. One of the major applications of satellite altimetry is to recover gravity information. In this paper, three methods (i.e., analytical and numerical inversion of the stokes formula, and the inverse Vening Meinesz formula) for deriving gravity anomalies from altimeter data are developed and studied in detail. The three formulae are implemented using fast Fourier transform (FFT) technique. And a series of modified spherical 2D FFT formula for the computation of gravity anomaly have been proposed in this step. Then they are compared and tested using synthetic data from an ultra high degree geopotential model MOD99c to degree and order 1440. The stability of the three approaches is investigated using simulated data, and many numerical tests are done to quantify the feature of the three approaches. Finally, the three formulae are employed to compute gravity anomalies over the South China Sea using geoidal undulations and deflections from Seasat, Geosat, ERS-1 and TOPEX/POSEIDON (T/P) satellite altimetry. And the estimated gravity anomalies are compared to marine gravity data from shipboard measurements in the studied area.

Keywords. Satellite altimetry, Marine gravity anomaly, FFT

1 Introduction

With the advent of satellite altimetry, the applications of satellite altimetry data have been extensively investigated. One of the major applications is to recover gravity information from satellite altimetry data. This has been done in the recent past (e.g., Knudsen and Andersen, 1997; Sandwell and Smith, 1997; Anderson and Knudsen, 1998; Hwang et al, 1998), and four corresponding altimetric-derived anomaly sets have been obtained. Rapp (1998) has made a detailed comparison of the above four altimetric-derived anomaly sets with the ship gravity data in the Gulf of California region. As we know, there exist three major approaches, i.e. the analytical and numerical inversion of the Stokes formula, and the inverse Vening Meinesz formula, for deriving gravity anomalies from altimeter data. The attention of the authors in this article is paid to the distinguishing characteristic of the three approaches. And the aim of this work is to find out which formula gains an advantage over the others in operational performance. First, the three formulae are implemented using fast Fourier transform (FFT) technique. And a series of modified spherical 2D FFT formula for the computation of gravity anomaly have been proposed in this step. Then they are compared and tested using synthetic data from an ultra high degree geopotential model MOD99c to degree and order 1440. The stability of the three approaches is investigated using simulated data, and many numerical tests are done to quantify the feature of the three approaches. Finally, the three formulae are employed to compute gravity anomalies over the South China Sea using geoidal undulations and deflections from Seasat, Geosat, ERS-1 and TOPEX/POSEIDON (T/P) satellite altimetry. And the estimated gravity anomalies are compared to marine gravity data from shipboard measurements in the studied area.

2 Mathematical Model

2.1 The Numerical Inversion of the Stokes Formula and its FFT Evaluation

According to Heiskanen and Moritz (1967), the well-known Stokes' integral equation can be expressed as

$$N = \frac{R}{4\pi\gamma} \iint \Delta g S(\psi) d\sigma \tag{1}$$

where R is a mean radius of the Earth; Δg is the free air gravity anomaly on a surface of the sphere of radius R; γ is the mean normal gravity of the

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Earth; ψ is the spherical distance between the computation point and the moving point of the integration; $S(\psi)$ is the Stokes kernel function, given by

$$S(\psi)=\frac{1}{s}-6s-4+10s^2-3(1-2s^2)\ln(s+s^2)$$

And the convolution form of Eq.(1) can be written as (Strang van Hees, 1990)

$$N(\varphi,\lambda)=\frac{R}{4\pi r}[\Delta g\cos\varphi]\times[S(\varphi,\lambda)] \tag{2}$$

The corresponding frequency-domain version of the above equation is

$$F[N(\varphi,\lambda)]=\frac{R}{4\pi\gamma}F[\Delta g\cos\varphi]\cdot F[S(\varphi,\lambda)] \tag{3}$$

The above equation can be used to recover gravity anomaly from geoidal undulations, which implies

$$\Delta g(\varphi,\lambda)=\frac{4\pi\gamma}{R\cos\varphi}F^{-1}\{\frac{F[N(\varphi,\lambda)]}{F[S(\varphi,\lambda)]}\} \tag{4}$$

Eq.(4) is just called as 2D spherical FFT evaluation of the numerical inversion of the Stokes formula.

According to Huang et al. (2000), to improve the conventional 2D spherical FFT evaluation, we suggest that Eq. (4) be rewritten as

$$\Delta g(\varphi,\lambda)=\frac{4\pi\gamma}{R\cos\varphi_M}F^{-1}\{\frac{F[N(\varphi,\lambda)]}{F[S(\varphi,\lambda)]}\} \tag{5}$$

where φ_M is the mean latitude of computation area. Although, at first appearance, Eq.(5) is only the approximation of Eq.(4), of practical importance is that, as mentioned in Huang et al. (2000), such modification will satisfy more closely the requirement of 2D FFT method. Consequently, it will not lead to the loss but gain in computation accuracy. This can be proved through numerical tests in the section 3.

2.2 The Analytical Inversion of the Stokes Formula and its FFT Evaluation

According to Molodenskii et al. (1962), the well-known Molodenskii equation (also known as inverse stokes' equation) can be expressed as

$$\Delta g_P=-\frac{\gamma}{R}N_P-\frac{\gamma}{16\pi R}\iint\frac{N_Q-N_P}{\sin^3(\psi/2)}d\sigma \tag{6}$$

Its frequency-domain version is

$$\Delta g_P=-\frac{\gamma}{R}N_P-\frac{\gamma}{4\pi R}\{F^{-1}[F[N(\varphi,\lambda)\cos\varphi] F[Z(\varphi,\lambda)]]-N_PF^{-1}[F[\cos\varphi]F[Z(\varphi,\lambda)]]\} \tag{7}$$

where

$$Z(\psi)=\frac{1}{4\sin^3\frac{\psi}{2}} \tag{8}$$

In the same way as Eq.(5), we suggest that Eq. (7) be rewritten as following form：

$$\Delta g_P=-\frac{\gamma}{R}N_P-\frac{\gamma\cos\varphi_M}{4\pi R}\{F^{-1}[F[N(\varphi,\lambda)]F[Z(\varphi,\lambda)]]-N_P\cos\varphi_M F^{-1}[F[1]F[Z(\varphi,\lambda)]]\} \tag{9}$$

Rewriting the kernel function $Z(\psi)$ as $Z(\psi)=Z(\varphi_P,\varphi_Q,\lambda_P-\lambda_Q)$, according to Haagmans et al. (1993), the 1D convolution form of Eq.(6) can be written as

$$\Delta g_P=-\frac{\gamma}{R}N_P-\frac{\gamma}{4\pi R}\int[N(\varphi,\lambda)\cos\varphi]*[Z(\varphi_P,\varphi,\lambda)]d\varphi+\frac{\gamma}{4\pi R}N_P\int[\cos\varphi]*[Z(\varphi_P,\varphi,\lambda)]d\varphi \tag{10}$$

The corresponding frequency-domain version of the above equation is

$$\Delta g_P=-\frac{\gamma}{R}N_P-\frac{\gamma}{4\pi R}F_1^{-1}\{\int F_1[N(\varphi,\lambda)\cos\varphi]F_1[Z(\varphi_P,\varphi,\lambda)]d\varphi+\frac{\gamma}{4\pi R}N_PF_1^{-1}\{\int F_1[\cos\varphi]F_1[Z(\varphi_P,\varphi,\lambda)]d\varphi\} \tag{11}$$

where F_1 and F_1^{-1} denote the 1D FFT operator and its inverse operator, respectively.

2.3 The Inverse Vening Meinesz Formula and its FFT Evaluation

According to Hwang(1998), ignoring the detailed derivation, we introduce directly the inverse Vening Meinesz formula as follows:

$$\Delta g(P) = \frac{\gamma}{4\pi}\iint_{\sigma} H'(\psi)(\xi\cos\alpha_{qp} + \eta\sin\alpha_{qp})d\sigma \qquad (12)$$

where α_{qp} represents the azimuth in the direction of move point to the computation point. ξ and η are the north-south component and west-east component of the deflection vector, respectively, and $H'(\psi)$ is the integral kernel function with following expression:

$$H'(\psi) = -\frac{\cos\frac{\psi}{2}}{2\sin^2\frac{\psi}{2}} + \frac{\cos\frac{\psi}{2}(3+2\sin\frac{\psi}{2})}{2\sin\frac{\psi}{2}(1+\sin\frac{\psi}{2})} \qquad (13)$$

The 2D and 1D spherical FFT evaluations of Eq.(12) can be written as, respectively

$$\begin{aligned}\Delta g_P = {} & \frac{\gamma}{4\pi}F^{-1}\{F[H'(\varphi,\lambda)\cos\alpha]F[\xi\cos\varphi] \\ & + F[H'(\varphi,\lambda)\sin\alpha]F[\eta\cos\varphi]\}\end{aligned} \qquad (14)$$

$$\begin{aligned}\Delta g_P = {} & \frac{\gamma}{4\pi}F_1^{-1}\{\sum_{\varphi_1}^{\varphi_n}[F_1[H'(\Delta\lambda)\cos\alpha]F_1[\xi\cos\varphi] \\ & + F_1[H'(\Delta\lambda)\sin\alpha]F_1[\eta\cos\varphi]]\}\end{aligned} \qquad (15)$$

Following the idea similar to Eq.(5), we suggest that the conventional 2D FFT Eq.(14) be rewritten as

$$\begin{aligned}\Delta g_P = {} & \frac{\gamma\cos\varphi_M}{4\pi}F^{-1}\{F[H'(\varphi,\lambda)\cos\alpha] \\ & F[\xi] + F[H'(\varphi,\lambda)\sin\alpha]F[\eta]\}\end{aligned} \qquad (16)$$

3 Numerical Tests

In order to demonstrate the improvement of the new 2D spherical FFT technique (new 2D-S for short) proposed here upon the conventional one (old 2D-S for short), first, several consistent sets of gravity, geoid and deflection data, generated from a spherical harmonic model, are used to make numerical tests of FFT gravity predictions in this section. In the sequel, a ultra high degree geopotential model called MOD99c up to degree and order 1440 (Huang, et al., 1999), developed by using altimeter-derived marine anomalies on a global scale (Hwang, et al., 1998) and the local data at high resolution in China, and EMG96 as start model, is used in different degree ranges, in which the lowest spherical harmonic coefficients have been removed, to simulate the typical prediction case. The generated gravity anomaies will be used as “ground truth” for comparing the results of FFT gravity predictions. Using the MOD99c geopotential model, several sets of geoidal undulations and deflections were produced on a 5′×5′ grid, and these data were then used as input to the spherical FFT softwares based on the mathematical models mentioned in the above section. The gravity anomalies computed by such way were compared with the called model gravity anomalies computed from the corresponding geopotential expansion and the rms of differences between them may be used to judge the accuracy of gravity predictions. A 55°×70° and three 10°×10° blocks located at different latitudes are chosen as our test areas in this study. The former corresponds to a 660×840 grid (554400 points). And each of the latter corresponds to a 120×120 grid (14400 points). The locations of the four test areas are: (A)0°~55°N，70°~140°E；(B)0°~10°N, 105°~115°E ； (C)35°~45°N, 80°~90°E ； (D)75°~85°N, 80°~90°E. A statistical result (rms) of different gravity field parameters in the degree range (361~1440) is summarized in Table 1.

Table 1. The statistics of gravity field quantity in the test areas / 1mGal= 1 cm/s^2

Parameter	A	B	C	D
Δg/mGal	17.15	5.56	29.02	5.17
N/m	0.211	0.073	0.356	0.066
ξ/arc-second	2.620	0.781	5.130	0.959
η/arc-second	2.488	0.876	3.311	0.570

Here gravity anomaly predictions have been carried out using the three approaches mentioned above in new and old 2D-S ways, respectively. The computation results are then compared to the corresponding "true" spherical harmonic values. For convenience, the numerical inversion of the Stokes formula is called N-I-Stokes for short, and the analytical inversion of the Stokes formula called A-I-Stokes, the inverse Vening Meinesz formula called I-Vening-Meinesz, respectively. Table 2 shows, first, the statistical results of the differences for the whole computation area A. It should be noted that, here, in all cases 100% zero-padding (Haagmans, et al., 1993) has been used in our computations.

Table 2. The numerical test results for different methods in area A/ mGal

Method		Max	Min	Mean	rms	sd
N-I Stokes	Old 2D-S	96.59	-101.36	0.004	2.983	2.983
	New 2D-S	69.58	-75.36	0.002	1.493	1.493
A-I Stokes	Old 2D-S	67.44	-90.77	0.003	4.441	4.441
	New 2D-S	61.06	-48.69	0.003	2.990	2.990
I-Vening Meisesz	Old 2D-S	76.06	-90.44	0.007	5.703	5.703
	New 2D-S	24.93	-25.74	0.005	1.227	1.227

From the comparisons of Table 2, it is can be seen that the results derived from the improved 2D spherical FFT in all the three approaches are superior to those obtained by the conventional 2D spherical FFT technique, indeed. And the accuracy improvements gained in the new 2D spherical FFT are very significant. This conclusion supports, again, the argument proposed in Huang et al. (2000) and verifies our claims.

We have made a similar test in area B, C and D, respectively. In order to evaluate the data edge effects on the estimated gravity information, we calculate, furthermore, the accuracy improvements gained when the results around the different width edges of the computation area were disregarded. To save space, Table 3 only lists the comparison results (RMS) corresponding to the 2D-S FFT N-I-Stokes, the 1D-S FFT A-I-Stokes and the 1D-S FFT I-Vening-Meinesz, respectively.

Table 3. The numerical test results for different methods in area A, B and C/ mGal

Disregarded width		0	30′	1°	1.5°	2°
Area B	N-I-Stokes	0.987	0.195	0.186	0.184	0.180
	A-I-Stokes	1.356	0.922	0.840	0.804	0.767
	I-Vening-Meisesz	0.772	0.453	0.425	0.419	0.405
Area C	N-I-Stokes	4.677	1.058	1.050	1.025	0.953
	A-I-Stokes	6.691	4.943	4.738	4.439	3.984
	I-Vening-Meisesz	3.502	2.493	2.476	2.350	2.098
Area D	N-I-Stokes	9.108	1.165	1.114	1.125	1.129
	A-I-Stokes	1.854	1.354	1.247	1.232	1.174
	I-Vening-Meisesz	1.154	0.837	0.826	0.825	0.861

It can be seen from Table 3 that although the data edge effects on the estimated gravity information from all the three approaches are outstanding, comparatively speaking, the effects on the N-I-Stokes is more significant. The results derived from the N-I-Stokes will not be practicable if ignoring the data edge effects. Furthermore, the prediction accuracies remain almost unchanged when the disregarded width around the edges of the computation area increases from 30´ to 2°. It means that disregarding a small part of the results (a limited width) around the edges is enough for eliminating the data edge effects on the estimated gravity anomalies。

4 Tests on the Stability of Computation Models

In the above section, we have made some numerical tests on the prediction accuracy of computation models using errorless input data. In this section, a noise sensitivity analysis of the three approaches is carried out in order to see how the estimated gravity anomalies from satellite altimetry data is affected by random and systematic noise.The instrumental noise for Geosat、ERS-1 and T/P is at the 2- to 5-cm level. In order to model this noise,random noise has been generated and added to the simulated geoidal undulations and deflections of the vertical from the MOD99c geopotential model. And then a similar numerical test has been made in area A. Table 4 lists the mean and rms values of the random noise effect on predicted gravity anomalies from geoidal undulations and deflections of the vertical derived from the MOD99c geopotential model with input noise. For example, when adding random noise with a standard deviation of 2.0 cm and a mean of 0.0 cm to the simulated geoidal undulations, the effect on the derived gravity anomalies is at the level of 5.4 mGal for 2D-S FFT N-I-Stokes, 3.3 mGal for 1D-S FFT A-I-Stokes, and 1.6 mGal for 1D-S FFT I-Vening-Meinesz in terms of rms. The I-Vening-Meinesz approach gives much better results than the other two I-Stokes methods. When the standard deviation of the random noise increases from 2.0 cm to 5.0 cm, the mean errors remain at almost the same level because the mean of the random noise is equal to 0.0 cm while the rms error increases by about three times. This shows clearly that the input noise is amplified by the inversion, which is expected since gravity is the vertical derivative of the geoid. And that is just why we are interested to have a detailed discussion on this kind of effect.

Table 4. test results of random noise effect / mGal

Noise level /m	N-I-Stokes		A-I-Stokes		I-Vening-Meinesz	
	Mean	rms	Mean	rms	Mean	rms
$\mu = 0$, $\sigma = 0.02$	0.001	5.377	-0.000	3.313	-0.000	1.602
$\mu = 0$, $\sigma = 0.05$	0.002	13.442	-0.000	8.282	-0.001	4.006
$\mu = 0$, $\sigma = 0.10$	0.005	26.883	-0.000	16.565	-0.001	8.012

In order to find out the effect of biases on the estimated gravity anomalies, systematic noise is added to the simulated geoidal undulations of the MOD99c geopotential model. Table 5 lists the mean and rms of the differences between the two sets of estimated gravity anomalies from two geoidal undulation sets, one with systematic noise, and the other errorless, for the two I-Stokes methods. This test is not done for the I-Vening-Meinesz method because systematic noise in geoidal undulations will be eliminated during the conversion of altimeter-measured geoidal undulations to deflections of the vertical in this kind of method. From Table 5, it can be seen that the effect of biases on the estimated gravity anomalies derived from 2D-S FFT N-I-Stokes is much stronger than those derived from 1D-S FFT A-I-Stokes. When adding a constant bias of 10 cm to the simulated geoidal undulations, the mean and rms of comparison differences for the A-I-Stokes are –0.02 and 0.02 mGal, while for the N-I-Stokes the corresponding values are –1.06 and 2.16 mGal, respectively. Furthermore, the mean and rms of comparison differences for the A-I-Stokes always change in a unified way when the constant bias increases from 10 cm to 100cm. It implies that the input bias affects only the mean value of the differences and not their standard deviation for the A-I-Stokes. This behavior is expected from the theory. A glance at Eq.(6) reveals that the bias affects only the first term and not the integral term. Thus, an input bias causes only an output bias and no output random errors.

Table 5. test results of systematic error influence/ mGal

Constant bias level /m	N-I-Stokes Mean	N-I-Stokes rms	A-I-Stokes Mean	A-I-Stokes rms
0.1	-1.063	2.158	-0.015	0.015
0.3	-3.189	6.475	-0.046	0.046
0.5	-5.315	10.791	-0.077	0.077
1.0	-10.630	21.582	-0.154	0.154

Up to now, all the simulation tests have been carried out using different inversion methods with errorless and erroneous input data. The results obtained above are definitely adequate for verifying the advantage of using the I-Vening-Meinesz approach over the other two methods. The advantage of this approach is embodied mainly in two aspects: first, the systematic noise does not affect the results derived from the I-Vening-Meinesz approach, and second, the I-Vening-Meinesz approach has given better results than the other two methods in limited simulation. Although the A-I-Stokes approach is insensitive to the systematic noise, it is affected strongly by the random noise. In addition, it is clear that the N-I-Stokes method is affected strongly not only by the random noise but also by the bias. Consequently, as stated previously, the I-Vening-Meinesz approach is recommended here for real altimetry application. A limited experimentation with real altimetry data in the section 5 will confirm the above analysis once again.

Generally, the accuracy of the recovered gravity anomalies depends on the following four aspects: a) data errors, which depends on the density, coverage and distribution of data, and its measurement accuracy; b) reference gravity field; c) modeling errors; d) numerical techniques (e.g., FFT techniques). In fact, the second and fourth aspects can be involved in the third aspect. According to the numerical tests in the section 3, for the I-Vening-Meinesz approach, the modeling errors can be improved to the level of only 2 mGal (see Table 2 and 3) when proper procedures are employed for the minimization or elimination of different effects. Additionally, as stated previously, the instrumental noise for Geosat、ERS-1 and T/P is at the 2 to 5 cm level. According to the tests on the stability of computation models, when considering the random noise of 5 cm as input data error, which is propagated into the results by the integral operator, the rms difference between the two sets of results derived from errorless and erroneous input data is about 4 mGal (see Table 4). Besides the measurement noise discussed above, it is clear that an additional error resulted from the conversion of altimeter-measured geoidal undulations to deflections of the vertical, which is necessary to the I-Vening-Meinesz approach, is affecting the results of the computations. The analysis in Huang et al. (2000) has shown that this conversion error is at the level of about 3 mGal. Summarizing all the effects from different aspects discussed above, the final accuracy of altimeter-derived gravity anomalies can be estimated theoretically as follows:

$$m = \pm\sqrt{2^2 + 4^2 + 3^2} = \pm 5.4\ (mGal) \qquad (17)$$

If a further consideration is taken for the errors of reference gravity field and other factors, it is maybe true that the quality of recovered gravity anomalies is poorer than we expected from Eq.(17). Further investigations have been done to verify the above claim in next section.

5 Practical Computations and Comparison to Shipborne Gravity

This section describes the application of the three inversion methods discussed above to real altimetry data over the South China Sea. The computation area is defined as: 2°~25°N; 105°~125°E. The MOD99a geopotential model (Huang et al., 1999) to degree 360 is used as the reference field. The altimeter data used are from Seasat, Geosat/ERM, Geosat/GM, ERS-1/35-day, ERS-1/GM and TOPEX/POSEIDON. The method of least squares collocation and the covariance functions of deflections derived by Hwang and Parsons (1995) are applied to grid the deflections of different azimuths into the north-south and west-east components at a 2′×2′ interval. A more detailed account of the altimeter data used has been given in Hwang et al. (1998). It is Prof. C.Hwang who has made all the gridded data available to us. The gridded geoidal undulations from the altimeter data are then used to compute the gravity anomalies over the chosen area by the 2D-S FFT N-I-Stokes and 1D-S FFT A-I-Stokes methods, and the two gridded geoid gradient components by the 1D-S

FFT I-Vening-Meinesz approach. Finally, three groups of computed results, which are corresponding to the three computation methods above and are named by group A, B and C, respectively, have been obtained in succession.

In order to evaluate the accuracy of the altimeter-derived gravity anomalies, the three groups of computed results are compared with the shipborne measurements of gravity anomalies requested from our department in the region 7° to 20°N, 110° to 120°E. The data came from numerous cruises dating from 1990 to 2000. It is believed that the accuracy of the ship anomaly is at the level of 1 to 3 mGal. Before comparison, the gravity anomalies from the altimeter-derived grid are interpolated to each ship anomaly location using weighted-mean method. The comparisons are made at all (155,808) stations. The statistics of the differences (altimeter minus ship anomaly) have been calculated in three cases as shown in Table 6.

Table 6. The statistics of differences between the altimeter and ship anomalies /mGal

Year		1990	1991	1993	1994	1995	1996	1997	1998	1999	2000
Number of data		3070	2545	3833	2532	2492	2407	13874	9681	56936	58438
Group A	Mean	-3.92	-3.32	-3.97	-3.81	-3.91	-3.88	-1.83	-1.79	-4.05	-2.25
	Rms	11.66	11.69	8.67	9.06	8.25	8.34	8.95	10.90	9.63	9.45
Group B	Mean	-3.31	-2.95	-3.61	-3.35	-3.51	-3.55	-1.18	-1.78	-3.71	-1.89
	Rms	8.42	8.66	5.89	6.58	5.97	5.45	5.76	7.34	6.75	6.73
Group C	Mean	-3.38	-2.90	-3.54	-3.01	-3.96	-3.26	-0.99	-1.89	-3.70	-1.64
	Rms	8.25	8.57	5.77	6.49	5.87	5.29	5.61	7.18	6.62	6.65

Table 7. The statistics of differences between the satellite and ship 5′×5′ anomalies /mGal

Year	Block No.	Max	Min	Mean	rms	Std.dev.
1990	883	30.77	-46.46	-4.08	9.44	8.52
1991	794	22.58	-49.18	-3.19	9.07	8.49
1993	1051	16.89	-42.94	-3.57	5.61	4.33
1994	852	28.15	-27.09	-3.15	6.12	5.24
1995	610	8.79	-26.06	-3.99	5.62	3.95
1996	635	17.69	-31.21	-3.43	5.35	4.10
1997	949	19.28	-16.97	-1.15	4.66	4.52
1998	640	24.09	-49.39	-1.88	6.57	6.29
1999	784	23.21	-24.49	-3.91	6.22	4.84
2000	920	18.16	-29.50	-1.50	5.03	4.80

Table 6 indicates that there exists a significant bias in all the comparison, which ranges from –1.0 mGal to –4.1 mGal. This bias is most likely associated with the bias values remaining in the ship gravity data or in the reference geopotential model. The group C anomalies show the best agreement with the ship data, with a rms of differences ranging from ±8.6 mGal to ±5.3 mGal, in contrast to from ±11.7 mGal to ±8.3 mGal for the group A anomalies, and ±8.7 mGal to ±5.5 mGal for the group B anomalies. This conclusion is consistent with the evaluation given in the section 4. The results obtained above are better than we expected, although the magnitude of the computation errors is slightly larger than the estimated value reported by Eq.(17).

We next consider the comparison of the mean block values between the altimeter-derived and ship gravity anomalies. In order to save space, only the results of comparison to the group C anomalies are given in the following. Table 7 lists, first, the statistics of the differences between the altimeter and ship 5′×5′ mean block anomalies. Compared to Table 6, no obvious change occurs in this new comparison due to the fact that the density of the marine gravity measurements is almost equal to the 5′ interval. Owing to the reason of the density of marine gravity surveying lines, here the comparisons of 15′×15′, 30′×30′ and 1°×1° mean block anomalies have been done only with the limited ship data obtained after 1997. Details of the comparison differences have been given in Table 8.

From Table 8, one sees that, if considering the existence of errors of marine gravity measurements, the accuracy of altimeter-derived 15′×15′, 30′×30′ and 1°×1° mean block anomalies can reach the level of 5 mGal, 4 mGal and 3 mGal, respectively. It is for the first time that we obtain this conclusion.

Table 8. The statistics of differences between the satellite and ship mean block anomalies /mGal

Grid	Year	Max	Min	Mean	rms	Std.dev.
15′×15′	1997	5.94	-15.36	-0.92	2.60	2.43
	1998	6.45	-12.45	-1.63	3.404	2.99
	1999	1.96	-11.78	-3.37	4.29	2.65
	2000	4.73	-7.64	-0.96	2.64	2.46
30′×30′	1997	2.42	-5.82	-0.561	1.77	1.68
	1998	2.10	-4.91	-1.132	2.11	1.78
	1999	-0.31	-6.17	-3.16	3.46	1.41
	2000	2.56	-4.41	-0.65	1.86	1.75
1°×1°	1997	1.71	-2.74	-0.19	1.18	1.16
	1998	0.57	-2.02	-0.76	1.20	0.93
	1999	-0.04	-4.25	-2.03	2.33	1.15
	2000	2.83	-1.47	-0.06	1.21	1.21

6 Conclusions

This article has concentrated on the evaluation of three inversion methods, i.e. the N-I-Stokes, the A-I-Stokes and the I-Vening-Meinesz, for recovering gravity anomalies from altimeter data. In the first part of this paper, a series of numerical tests have been carried out to show the improvement of the new 2D-S FFT technique proposed here on the old one, and to investigate the stability of the three inversion methods using simulated data. A couple of important conclusions can be drawn at least from the studies and tests above. First, the use of the new 2D-S FFT approach has resulted in a significant improvement over the conventional one, indeed. The second conclusion is that the sensitivity of the I-Vening-Meinesz formula to random and systematic noise is lower than those of the N-I-Stokes and the A-I-Stokes methods.

In the second part of this paper, the discussions have considered only one aspect of comparing altimeter-derived anomalies with ship data. These comparisons lead to the following three important conclusions. First, a significant bias is seen in all the altimeter-derived/ship anomaly comparisons. The cause of this may be related to the errors remaining in the ship gravity data or in the reference geopotential model. The second conclusion is that the anomalies set of the I-Vening-Meinesz approach gives the best agreement with the ship data, in which the deflections of the vertical from the altimeter data are used as input information, instead of the sea surface heights. Finally, we have found out for the first time that the accuracy of altimeter-derived 15′×15′, 30′×30′ and 1°×1° mean block anomalies can reach the level of 5 mGal, 4 mGal and 3 mGal, respectively.

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References

Anderson, O., and P. Knudsen (1998). Global Marine Gravity Field from the ERS-1 and GEOSAT Geodetic Mission Altimetry, J Geophys Res., 103, pp. 8129-8137.

Haagmans, R., E. D. Min and M. V. Gelderen (1993). Fast Evaluation of Convolution Integrals on the Sphere Using 1DFFT, and a Comparison with Existing Methods for Stokes' Integral, Manuscripta Geodaetica, 18, pp. 227-241.

Heiskanen, W., and H. Moritz (1967). Physical Geodesy, New York, W.H. Freeman.

Huang, M. T., G. J. Zhai and Z. Guan (2000). Comments on FFT Technique in Spectral Geoid Determination, Acta Geodaetica et Cartographica Sinica, 29, pp. 124-131.
Huang, M. T., G. J. Zhai and Z. Guan (1999). Method of Extending High Degree Geopotential Model Using Altimeter-derived Marine Gravity Anomalies and the Local Data in China, Symposium for the Fortieth Anniversary of Chinese Society of Geodesy, Photogrammetry and Cartography. Beijing: Chinese Society of Geodesy, Photogrammetry and Cartography, pp. 203-210.
Hwang, C., and B. Parsons (1995). Gravity Anomalies Derived from Seasat, Geosat, ERS-1 and TOPEX/POSEIDON Altimetry and Ship Gravity: A Case-study over the Reykjanes Ridge, Geophys J Int, 122, pp. 511-568.
Hwang, C. (1998). Inverse Vening Meinesz Formula and Deflection-geoid Formula: Applications to the Predictions of Gravity and Geoid over the South China Sea, J of Geod, 72, pp. 304-312.
Hwang, C., E. C. Kao and B. Parsons (1998). Global Derivation of Marine Gravity Anomalies from SEASAT, Geosat, ERS-1 and TOPEX/POSEIDON Altimeter Data, Geophys J Int, 134, pp. 449-459.
Knudsen, P., and O. Andersen (1997). Improved Recovery of the Global Marine Gravity Field from the GEOSAT and the ERS-1 Geodetic Mission, In: Segawa J., Fujimoto H., Okubo S.(Eds), IAG Symposia 117, pp. 461-469.
Molodenskii, M. S., V. F. Eremeev and M. I. Yurkina (1962). Methods for Study of the External Gravitational Field and Figure of the Earth, Moscow: Works of Central Research Institute of Geodesy, Aerial Photography and cartography, No.131.
Rapp, R. H. (1998). Comparison of Altimeter-derived and Ship Gravity Anomalies in the Vicinity of the Gulf of California, Mar Geod, 21, pp. 245-259.
Sandwell, D., and W. Smith (1997). Marine Gravity Anomaly from Geosat and ERS-1 Satellite Altimetry, J Geophys Res, 102, pp. 10039-10054.
Strang, V. G. (1990). Stokes Formula Using Fast Fourier Techniques, Manuscripta Geodaetica, 15, pp. 235-239